GCSE MATHEMATICS · HIGHER

Targeting Grade 9

Paper 9 · Non-calculator · 50 minutes · 40 marks

Name ..... **Date** .....

Answer **all** questions. Show all your working: answers without working may not get full marks. Diagrams are **not** accurately drawn. You must **not** use a calculator.

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1 Solve

$$\left(\sqrt{2}\right)^{x} = 32^{x-3}$$

$x =$ .....

(Total for Question 1 is 3 marks)

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2 Prove that the product of any two consecutive even numbers is always one less than the square of an odd number.

(Total for Question 2 is 3 marks)

3 The first two terms of a geometric sequence are

$$\left(1+\sqrt{3}\right) \quad \text{and} \quad \left(3+\sqrt{3}\right)$$

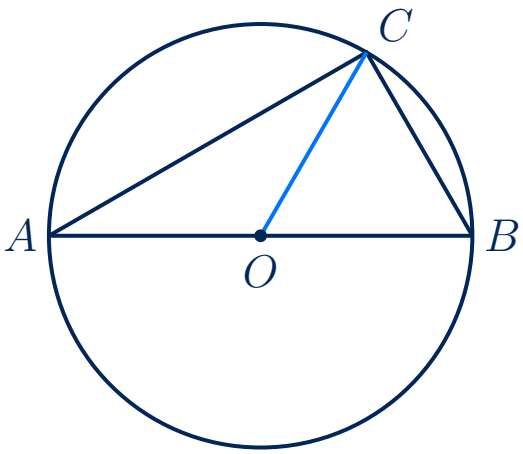
Work out the 5th term of the sequence.

Give your answer in the form $a+b\sqrt{3}$ where a and b are integers.

.....

(Total for Question 3 is 4 marks)

4



A, B and C are points on a circle, centre O.

AB is a diameter of the circle.

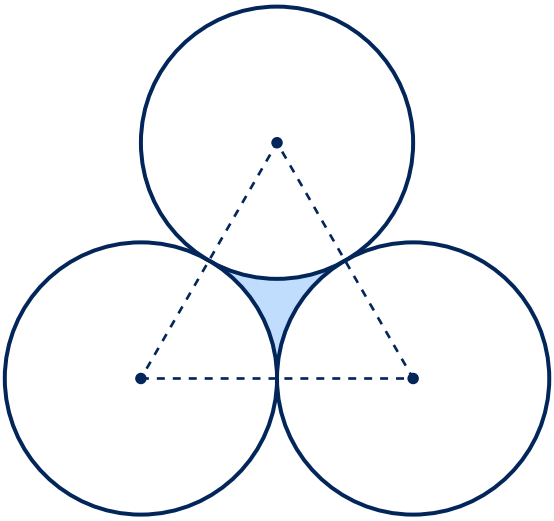
Prove that angle $ACB = 90°$

You must **not** use the circle theorem that the angle in a semicircle is $90°$

Give a reason for each stage of your working.

(Total for Question 4 is 4 marks)

5



The diagram shows three circles.

Each circle has a radius of 2 cm.

Each circle touches the other two circles.

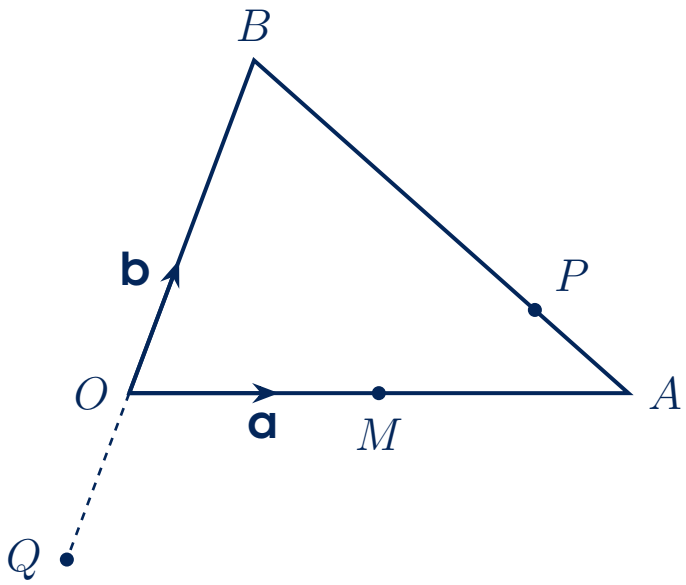
Work out the exact area of the shaded region enclosed between the three circles.

Give your answer in the form $a\sqrt{3} - b\pi$ where a and b are integers.

cm^2 .....

(Total for Question 5 is 4 marks)

6



OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

M is the midpoint of OA.

P is the point on AB such that $AP : PB = 1 : 3$

Q is the point such that $\overrightarrow{OQ} = -\frac{1}{2}\mathbf{b}$

Prove that M, P and Q lie on a straight line.

(Total for Question 6 is 5 marks)

7 The functions f and g are such that

$$f(x) = 2x + 1 \qquad g(x) = x^2$$

(a) Solve $fg(x) = gf(x)$

.....

(3)

(b) Find the set of values of x for which $fg(x) > gf(x)$

.....

(2)

(Total for Question 7 is 5 marks)

8 Solve the simultaneous equations

$$2x^2 + y^2 = 19$$

$$x + y = 4$$

. . . . . .

. . . . . .

(Total for Question 8 is 6 marks)

9 A bag contains 3 red counters and n green counters.

Two counters are taken at random from the bag, without replacement.

The probability that the two counters are different colours is $\frac{1}{2}$

(a) Show that $n^2 - 7n + 6 = 0$

(3)

(b) There are two possible values of n.

For each value of n, work out the probability that both counters are red.

.....

.....

(3)

(Total for Question 9 is 6 marks)

TOTAL FOR PAPER IS 40 MARKS