GCSE MATHEMATICS · HIGHER

Targeting Grade 9

Paper 1 · Non-calculator · 50 minutes · 40 marks

Name ..... **Date** .....

Answer **all** questions. Show all your working: answers without working may not get full marks. Diagrams are **not** accurately drawn. You must **not** use a calculator.

---

1 Prove that the product of any four consecutive integers is always one less than a square number.

(Total for Question 1 is 4 marks)

---

2 A box contains 12 counters.

x of the counters are red. The rest of the counters are yellow.

Two counters are taken at random from the box, without replacement.

Given that **at least one** of the counters is red, the probability that **both** counters are red is $\frac{2}{9}$

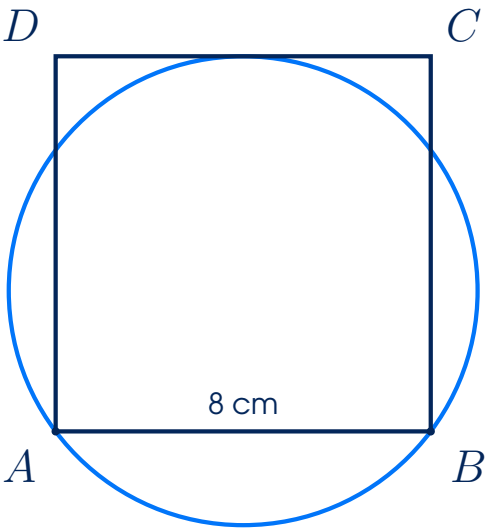
Work out the value of x.

You must show all your working.

$x =$ .....

(Total for Question 2 is 5 marks)

3



$ABCD$ is a square with sides of length 8 cm.

A circle passes through the points A and B.

The side CD is a tangent to the circle.

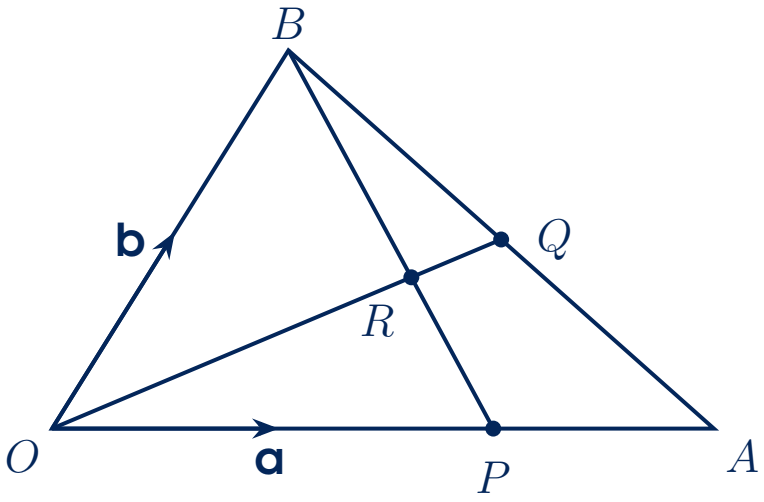
Work out the ratio area of the circle : area of the square

Give your answer in the form $k\pi : n$ where k and n are integers.

.....

(Total for Question 3 is 5 marks)

4



OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

P is the point on OA such that $OP : PA = 2 : 1$

Q is the midpoint of AB.

The lines OQ and BP intersect at the point R.

Work out the ratio $OR : RQ$

You must show all your working.

. . . . . .

(Total for Question 4 is 5 marks)

5 The function f is such that

$$f(x) = x^2 + 4x \qquad x \geqslant -2$$

(a) Find $f^{-1}(x)$

$f^{-1}(x) =$ .....

(3)

(b) Solve $ff(x) = 12$

Give your answer in the form $a + \sqrt{b}$ where a and b are integers.

$x =$ .....

(4)

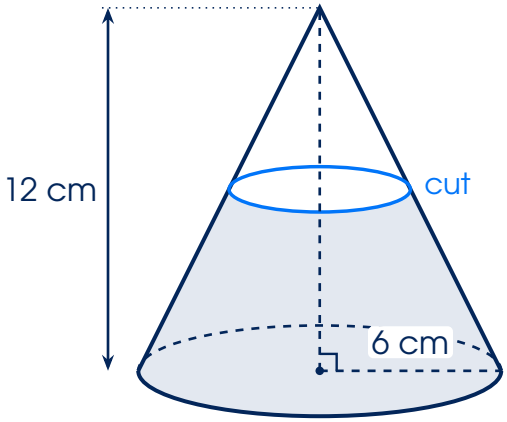
(Total for Question 5 is 7 marks)

6 A solid cone has base radius 6 cm and perpendicular height 12 cm.

The cone is cut by a plane parallel to its base.
The small cone above the cut is removed, leaving a frustum.

The volume of the frustum is $\frac{7}{8}$ of the volume of the original cone.

Work out the total surface area of the frustum.
Give your answer in the form $\pi(a + b\sqrt{5})$ where a and b are integers.



Curved surface area of a cone $= \pi rl$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

cm^2 .....

(Total for Question 6 is 6 marks)

7 C is the circle with equation $x^2 + y^2 = 25$

O is the centre of C.

P is the point with coordinates $(7, 1)$

The two tangents to C from P touch C at the points A and B.

(a) Work out the coordinates of A and the coordinates of B.

You must show all your working.

A (,)

B (,)

(6)

(b) Work out the area of the quadrilateral $OAPB$.

.....

(2)

(Total for Question 7 is 8 marks)

TOTAL FOR PAPER IS 40 MARKS