GCSE MATHEMATICS · HIGHER

Targeting Grade 9

Paper 1 · Mark scheme · Non-calculator · 40 marks

M method mark **P** process mark **A** accuracy mark (depends on the M or P marks before it)
B independent mark **C** communication mark **ft** follow through **oe** or equivalent

Q	Working	Answer	Mark	Notes
1	Integers $n,\ n+1,\ n+2,\ n+3$ $n(n+3) = n^2+3n,$ $(n+1)(n+2) = n^2+3n+2$		M1	Pairs the outer and inner terms (or expands fully to $n^4+6n^3+11n^2+6n$)
	Let $m = n^2+3n$: product $= m(m+2) = m^2+2m$		M1	
	$m^2+2m+1 = (m+1)^2 =$ $(n^2+3n+1)^2$		A1	
			C1	Conclusion: product $= (n^2+3n+1)^2-1,$ one less than a square number

Q	Working	Answer	Mark	Notes
2	$P(\text{both red}) = \dfrac{x(x-1)}{132}$		P1	
	$P(\text{at least one red}) = 1 -$ $\dfrac{(12-x)(11-x)}{132} = \dfrac{x(23-x)}{132}$		P1	
	$\dfrac{x(x-1)}{x(23-x)} = \dfrac{2}{9}$		P1	Conditional probability: $P(\text{both}) \div P(\text{at least one})$
	$9x-9 = 46-2x$		P1	Reduces to a linear equation
		$x = 5$	A1	

Q	Working	Answer	Mark	Notes
3	Centre lies on the perpendicular bisector of AB; distance from centre to CD is r, so distance from centre to AB is $8-r$		P1	Or coordinates: $A(0,0)$, $B(8,0)$, centre $(4,\ 8-r)$
	$r^2 = 4^2 + (8-r)^2$		P1	Pythagoras in the triangle with half of AB
	$r^2 = 16 + 64 - 16r + r^2 \Rightarrow r = 5$		A1	
	Area of circle $= 25\pi$, area of square $= 64$		P1	ft their r
		$25\pi : 64$	A1	

Q	Working	Answer	Mark	Notes
4	$\overrightarrow{OQ} = \frac{1}{2}(\mathbf{a}+\mathbf{b})$, $\overrightarrow{BP} = \frac{2}{3}\mathbf{a} - \mathbf{b}$		P1	Either vector correct
	$\overrightarrow{OR} = \frac{\lambda}{2}(\mathbf{a}+\mathbf{b})$ and $\overrightarrow{OR} = \mathbf{b} + \mu\left(\frac{2}{3}\mathbf{a} - \mathbf{b}\right)$		P1	Two expressions for $\overrightarrow{OR}$
	$\mathbf{a}$: $\frac{\lambda}{2} = \frac{2}{3}\mu$; $\mathbf{b}$: $\frac{\lambda}{2} = 1 - \mu$		P1	Equates coefficients
	$\mu = \frac{3}{5}$, $\lambda = \frac{4}{5}$ so $\overrightarrow{OR} = \frac{4}{5}\overrightarrow{OQ}$		P1	
		$4 : 1$	A1	

Q	Working	Answer	Mark	Notes
5(a)	$y = (x+2)^2 - 4$		M1	Completes the square
	$x = (y+2)^2 - 4 \Rightarrow y + 2 = \sqrt{x+4}$		M1	Rearranges (positive root as $x \geqslant -2$)
		$\sqrt{x+4} - 2$	A1	
5(b)	$\big(\mathrm{f}(x)\big)^2 + 4\mathrm{f}(x) = 12$		M1	Or $\mathrm{f}(x) = \mathrm{f}^{-1}(12) = \sqrt{16} - 2$
	$(\mathrm{f}(x)+6)(\mathrm{f}(x)-2) = 0 \Rightarrow \mathrm{f}(x) = 2$		M1	Rejects -6 (minimum value of f is -4)
	$x^2 + 4x - 2 = 0 \Rightarrow x = -2 \pm \sqrt{6}$		M1	
		$-2 + \sqrt{6}$	A1	Rejects $-2 - \sqrt{6}$ as $x \geqslant -2$

Q	Working	Answer	Mark	Notes
6	Volume of small cone is $\frac{1}{8}$ of the cone, so length scale factor $\sqrt[3]{\frac{1}{8}} = \frac{1}{2}$		P1	
	Small cone: radius 3, height 6		P1	
	Slant heights: $\sqrt{6^2 + 12^2} = 6\sqrt{5}$ and $3\sqrt{5}$		P1	
	Curved surface: $\pi \times 6 \times 6\sqrt{5} - \pi \times 3 \times 3\sqrt{5} = 27\sqrt{5}\,\pi$		P1	
	Circles: $36\pi + 9\pi = 45\pi$		P1	Both circular faces included
		$\pi(45 + 27\sqrt{5})$	A1	

Q	Working	Answer	Mark	Notes
7(a)	$OA \perp AP$ (tangent is perpendicular to the radius). Let $A = (x, y)$		M1	
	$\frac{y}{x} \times \frac{y-1}{x-7} = -1 \Rightarrow$ $x^2 + y^2 - 7x - y = 0$		M1	Product of gradients $= -1$ (or Pythagoras: $OA^2 + AP^2 = OP^2$)
	Using $x^2 + y^2 = 25$: $7x + y = 25$		M1	
	$x^2 + (25 - 7x)^2 = 25 \Rightarrow$ $50x^2 - 350x + 600 = 0$ $x^2 - 7x + 12 = 0$		M1	Substitutes and forms a 3-term quadratic
	$x = 3$ or $x = 4$		A1	
		$(3, 4)$ and $(4, -3)$	A1	Either order
7(b)	$AP = \sqrt{4^2 + 3^2} = 5$ (or $OP = \sqrt{50}$, $AP = \sqrt{50 - 25}$)		M1	
	$2 \times \frac{1}{2} \times 5 \times 5$	25	A1	
		Total	**40**	