VOLUME OF CONES, PYRAMIDS AND SPHERES

BUILDING BLOCKS

NAME _____ CLASS _____ DATE _____

KEY IDEA

A cone and a cylinder have the same base and height.
It takes 3 cones of water to fill the cylinder.

So a cone holds $\frac{1}{3}$ of the cylinder.

A pyramid holds $\frac{1}{3}$ of a prism with the same base and height.



1 Name each 3-D shape. Use the words in the box.

cone | cylinder | sphere | square-based pyramid | triangular prism



_____ _____ _____ _____ _____

2 Use the bar models to find one third of each number.

a)



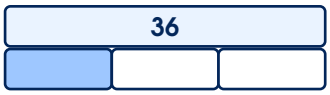
$\frac{1}{3}$ of 12 = ☐

b)



$\frac{1}{3}$ of 21 = ☐

c)

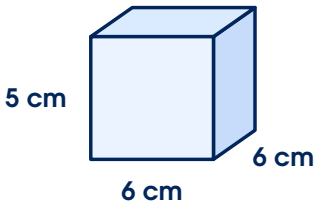


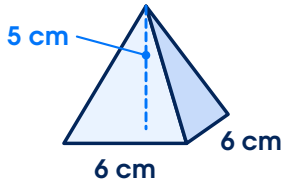
$\frac{1}{3}$ of 36 = ☐

3 Complete the table of cube numbers. r^3 means r × r × r ("r cubed").

r	2	3	4	5
r × r × r	2 × 2 × 2			
r^3	8			

4 The cuboid and the pyramid have the same base and height. Find both volumes.





Cuboid: ☐ × ☐ × ☐

= ☐ cm^3

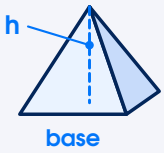
Pyramid: ☐ ÷ 3 = ☐ cm^3

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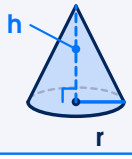
SUPPORT

NAME _____ CLASS _____ DATE _____

KEY IDEA



$V = \frac{1}{3} \times \text{base area} \times h$



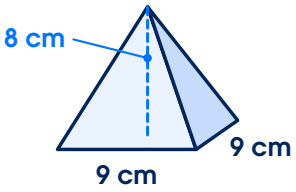
$V = \frac{1}{3} \times \pi \times r^2 \times h$



$V = \frac{4}{3} \times \pi \times r^3$

h is the perpendicular height: straight up from the centre of the base to the top.

1 Work out the volume of this square-based pyramid.

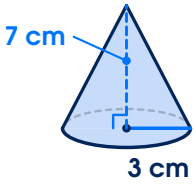


Base area = ☐ × ☐ = ☐ cm^2

$V = \frac{1}{3} \times$ ☐ × ☐ = ☐ cm^3

The base is a square.

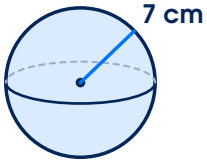
2 Work out the volume of this cone.



$V = \frac{1}{3} \times \pi \times$ ☐^{2} × ☐ = ☐ cm^3

Put r and h in the boxes. Use the π button. Round to 1 d.p.

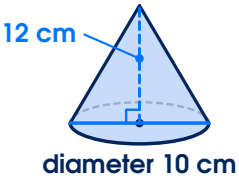
3 Work out the volume of this sphere.



$V = \frac{4}{3} \times \pi \times$ ☐^{3} = ☐ cm^3

r^3 means r × r × r. Round to 1 decimal place.

4 Work out the volume of this cone.



Radius r = ☐ ÷ 2 = ☐ cm

$V = \frac{1}{3} \times \pi \times$ ☐^{2} × ☐ = ☐ cm^3

Halve the diameter to find the radius first.

VOLUME OF CONES, PYRAMIDS AND SPHERES

CORE

NAME _____ CLASS _____ DATE _____

KEY IDEA

Pyramid: $V = \frac{1}{3} \times \text{base area} \times h$

Cone: $V = \frac{1}{3} \times \pi \times r^2 \times h$

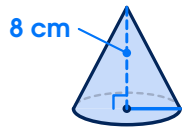
Sphere: $V = \frac{4}{3} \times \pi \times r^3$

h is the perpendicular height, not the slant height. Check if you are given a diameter.

FLUENCY

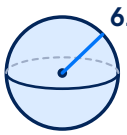
1 Work out the volume of each solid. Give your answers to 1 decimal place.

a)



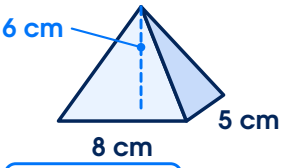
_____ cm³

b)



_____ cm³

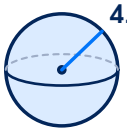
c)



_____ cm³

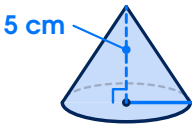
2 Work out the volume of each solid. Give your answers in terms of π.

a)



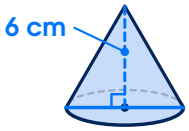
_____ cm³

b)



_____ cm³

c)

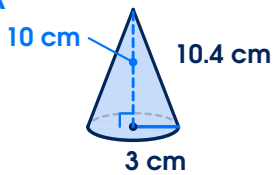


_____ cm³

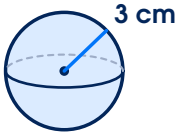
REASONING

3 Write the letters in order of volume, smallest first. Show your working.

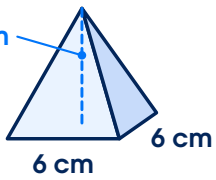
A



B



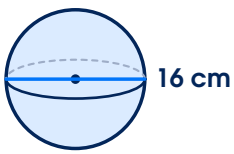
C



smallest → largest

_____ _____ _____

4 Sam finds the volume of a sphere of diameter 16 cm. Explain his mistake.



Sam's working

$V = \frac{4}{3} \times \pi \times 16^3 = 17157.3 \text{ cm}^3$

Correct volume (1 d.p.)

_____ cm³

VOLUME OF CONES, PYRAMIDS AND SPHERES

GREATER DEPTH

NAME _____ CLASS _____ DATE _____

REASONING

1 Is each statement always, sometimes or never true? Give a reason for each.

a) A cone is one third of the volume of a cylinder. ALWAYS SOMETIMES NEVER

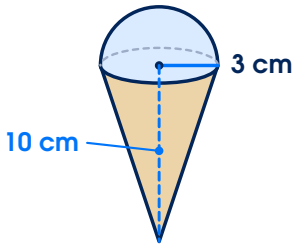
b) Doubling the radius of a sphere doubles its volume. ALWAYS SOMETIMES NEVER

c) A pyramid holds less than a cube with the same base. ALWAYS SOMETIMES NEVER

d) A cone's slant height is longer than its height. ALWAYS SOMETIMES NEVER

PROBLEM SOLVING

2 An ice cream is a full cone with a hemisphere on top. Both have radius 3 cm.



a) Total volume, in terms of π _____ cm³

b) How many ice creams can be made from 1 litre (1000 cm³)?

_____

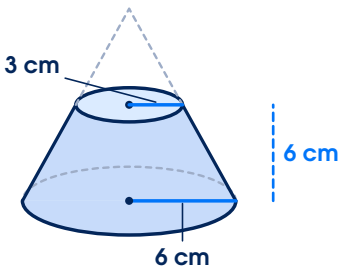
3 Work backwards from the volume to find each radius. Show your working.

a) A sphere has a volume of 4500π cm³. Its radius is _____ cm

b) A cone has a volume of 150π cm³ and a height of 18 cm. Its radius is _____ cm

★ CHALLENGE FOR ALL

4 This solid is a frustum.



A cone of radius 6 cm and height 12 cm has a cone of radius 3 cm and height 6 cm cut off the top. The piece left is a frustum.

a) Volume of the frustum, in terms of π _____ cm³

b) Explain why the frustum is not half of the big cone, even though the cut is halfway up.

VOLUME OF CONES, PYRAMIDS AND SPHERES – ANSWERS

ANSWERS

BUILDING BLOCKS

1 sphere · square-based pyramid · cone · triangular prism · cylinder
2 a) 4 b) 7 c) 12
3 $3 \times 3 \times 3 = 27$ · $4 \times 4 \times 4 = 64$ · $5 \times 5 \times 5 = 125$
4 Cuboid $6 \times 6 \times 5 = 180$ cm³ · pyramid $180 \div 3 = 60$ cm³

SUPPORT

1 $9 \times 9 = 81$ cm² · $1/3 \times 81 \times 8 = 216$ cm³
2 $1/3 \times \pi \times 3^2 \times 7 = 21\pi = 66.0$ cm³
3 $4/3 \times \pi \times 7^3 = 4/3 \times \pi \times 343 = 1436.8$ cm³
4 $r = 10 \div 2 = 5$ cm · $1/3 \times \pi \times 5^2 \times 12 = 100\pi = 314.2$ cm³

CORE

1 a) $1/3 \times \pi \times 25 \times 8 = 209.4$ cm³ b) $4/3 \times \pi \times 6.5^3 = 1150.3$ cm³ c) $1/3 \times 8 \times 5 \times 6 = 80$ cm³
2 a) $4/3 \times \pi \times 4.5^3 = 121.5\pi$ cm³ b) $1/3 \times \pi \times 36 \times 5 = 60\pi$ cm³ c) $r = 4$: $1/3 \times \pi \times 16 \times 6 = 32\pi$ cm³
3 A $= 30\pi = 94.2$ cm³ (use 10 cm, not the slant 10.4 cm) · B $= 36\pi = 113.1$ cm³ · C $= 1/3 \times 36 \times 9 = 108$ cm³. Order: A, C, B
4 He used the diameter as the radius. $r = 8$: $4/3 \times \pi \times 512 = 2144.7$ cm³ (his answer is 8 times too big).

GREATER DEPTH

1 a) sometimes (only for a cylinder with the same base and height) b) never (the volume is multiplied by 8) c) sometimes (a pyramid more than 3 times as tall as the cube holds more) d) always (the slant height is the hypotenuse of a right-angled triangle with the height as one side)
2 a) cone $1/3 \times \pi \times 9 \times 10 = 30\pi$, hemisphere $1/2 \times 4/3 \times \pi \times 27 = 18\pi$, total 48π cm³ (150.8 cm³) b) $1000 \div 150.8 = 6.6...$, so 6 ice creams
3 a) $4/3 \times r^3 = 4500$, $r^3 = 3375$, $r = 15$ cm b) $1/3 \times r^2 \times 18 = 150$, $6r^2 = 150$, $r^2 = 25$, $r = 5$ cm
4 a) $1/3 \times \pi \times 36 \times 12 - 1/3 \times \pi \times 9 \times 6 = 144\pi - 18\pi = 126\pi$ cm³ b) The small cone is only 1/8 of the big cone (radius and height both halve), so the frustum is 7/8 of the big cone, not 1/2.