HIGHER ONLY

HIGHER TIER

Name ..... Class .....

A20: Iteration

Solve problems using iteration and change of sign

20 marks · 25 minutes · Calculator allowed

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1) A box with a square base of side x cm and height $(x + 2)$ cm has volume 50 cm^3. Show that x lies between 3 and 4. Use $x_{n+1} = \sqrt{\dfrac{50}{x_n + 2}}$ with $x_0 = 3$ to find x to 2 decimal places.

..... cm

[Question 1: 5 marks]

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2) A population follows $P_{n+1} = 0.95P_n + 100$, with $P_0 = 5000$. After how many years is the population first below 4000?

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[Question 2: 3 marks]

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3) Show that $x^3 - 6x + 2 = 0$ has a solution between 0 and 1. Use $x_{n+1} = \frac{x_n^3 + 2}{6}$ with $x_0 = 0$ to find it to 3 decimal places.

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[Question 3: 4 marks]

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4) Ben uses $x_{n+1} = 6 - \frac{2}{x_n}$ with $x_0 = 1$. Find the value the iteration approaches to 3 decimal places, and the equation it solves.

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[Question 4: 4 marks]

5) Rearrange $x^2 = 5x - 3$ into the form $x = \sqrt{ax + b}$. Use $x_0 = 4$ to find a solution to 3 decimal places.

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[Question 5: 4 marks]

End of paper – 20 marks in total

Answers and brief mark scheme

Q	Answer	Method / notes
1	3.12	$f(3) = 45 < 50$, $f(4) = 96 > 50$ (P1); iterates 3.1623, 3.1122, 3.1274 (P1, P1); further values (P1); 3.12 (A1).
2	8	Iterates 4850, 4707.5, ... (P1, P1); $P_8 = 3990.3$ (A1).
3	0.340	$f(0) = 2$, $f(1) = -3$ (P1); 0.3333, 0.3395, 0.3399 (P1, P1); A1.
4	5.646; $x^2 - 6x + 2 = 0$	Iterates 4, 5.5, 5.636 (P1, P1); 5.646 (A1); equation (A1).
5	4.303	$x = \sqrt{5x - 3}$ (P1); iterates (P1, P1); A1.