FOUNDATION & HIGHER

HIGHER EXTENDS

Name ..... Class .....

A6: Algebraic proof

Use algebra to solve problems and prove results

20 marks · 25 minutes · Non-calculator

1) Show that the sum of any four consecutive integers is always 2 more than a multiple of 4

[Question 1: 3 marks]

2) Show that $n^2 + 3n + 2$ is always even for any integer n.

[Question 2: 3 marks]

3) Prove that the difference between the squares of two consecutive odd numbers is always a multiple of 8

[Question 3: 3 marks]

4) Think of a number n. Square it, add $2n$, add 1, take the positive square root, then subtract n. Prove that the answer is always 1 (for $n > 0$).

[Question 4: 3 marks]

5) $(x + a)(x + b) \equiv x^2 + 7x + 10$ and $a > b$. Find a and b.

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[Question 5: 2 marks]

6) Prove that the product of three consecutive integers is always a multiple of 6

[Question 6: 3 marks]

7) Use an algebraic identity to work out $37^2 - 33^2$ without a calculator.

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[Question 7: 3 marks]

End of paper – 20 marks in total

Answers and brief mark scheme

Q	Answer	Method / notes
1	Proof	$n+(n+1)+(n+2)+(n+3) = 4n+6$ (P1); $= 4(n+1)+2$ (P1); conclusion (C1).
2	Proof	$(n + 1)(n + 2)$ (P1); consecutive integers, one is even (P1); C1.
3	Proof	$(2n + 3)^2 - (2n + 1)^2$ (P1); $= 8n + 8$ (P1); $8(n + 1)$ (C1).
4	Proof	$n^2 + 2n + 1 = (n + 1)^2$ (P1); root $n + 1$ (P1); $-n = 1$ (C1).
5	$a = 5, b = 2$	$a + b = 7, ab = 10$ (P1); A1.
6	Proof	Among three consecutive integers one is a multiple of 3 (P1); at least one is even (P1); so a multiple of 6 (C1).
7	280	$(37 - 33)(37 + 33)$ (P1, P1); 4×70 (A1).