FOUNDATION & HIGHER

HIGHER EXTENDS

Name ..... Class .....

G25: Vector proof

Use vectors to construct geometric arguments and proofs

20 marks · 25 minutes · Non-calculator

1) Show that $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$ are parallel.

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[Question 1: 2 marks]

2) $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. M is the midpoint of AB. Prove that $\overrightarrow{OM} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$

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[Question 2: 2 marks]

3) $OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. Prove that its diagonals bisect each other.

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[Question 3: 3 marks]

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4) Explain why $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ when $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

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[Question 4: 2 marks]

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5) Kim says $|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$ always. Use $\mathbf{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ to show she is wrong.

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[Question 5: 2 marks]

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6) $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = 3\mathbf{a}$. Explain why O, A and B lie on a straight line.

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[Question 6: 2 marks]

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7) $OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of AB. Show that $\overrightarrow{OM} = \mathbf{a} + \frac{1}{2}\mathbf{c}$

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[Question 7: 2 marks]

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8) In triangle OAB, P and Q are the midpoints of OA and OB. Prove that PQ is parallel to AB and half its length.

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[Question 8: 3 marks]

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9) Explain why $k\mathbf{a}$ is parallel to $\mathbf{a}$ for any non-zero number k.

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[Question 9: 2 marks]

End of paper – 20 marks in total

Answers and brief mark scheme

Q	Answer	Method / notes
1	Shown	$\begin{pmatrix}4\\6\end{pmatrix} = 2\begin{pmatrix}2\\3\end{pmatrix}$ (M1); C1.
2	Proof	$\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$ (M1); C1.
3	Proof	Midpoint of OB: $\frac{1}{2}(\mathbf{a} + \mathbf{c})$ (M1); midpoint of AC: $\mathbf{a} + \frac{1}{2}(\mathbf{c} - \mathbf{a})$ (M1); equal (C1).
4	Explanation	$A \to O \to B$ is $-\mathbf{a} + \mathbf{b}$ (C2).
5	Shown	$\lvert\mathbf{a} + \mathbf{b}\rvert = 5$ (M1); $3 + 4 = 7$ (C1).
6	Explanation	$\overrightarrow{OB}$ is a multiple of $\overrightarrow{OA}$ (C1); share point O (C1).
7	Shown	$\overrightarrow{AB} = \mathbf{c}$ (M1); C1.
8	Proof	$\overrightarrow{PQ} = \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$ (M1); $= \frac{1}{2}(\mathbf{b} - \mathbf{a})$ (M1); $= \frac{1}{2}\overrightarrow{AB}$ (C1).
9	Explanation	Same direction (or opposite), scaled length (C2).