HIGHER ONLY
HIGHER TIER

Name ..... Class .....

# G23: Area of a triangle

*Reason with Area* = $\frac{1}{2}ab\sin C$

20 marks · 25 minutes · Calculator allowed

**1)** Show that $\frac{1}{2}ab\sin C$ is the same as $\frac{1}{2} \times$ base $\times$ height, using the height $h = b\sin C$.

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**[Question 1: 3 marks]**

**2)** Show that a triangle with sides 6 cm and 8 cm and an included angle of 30° has area 12 $cm^2$.

**[Question 2: 2 marks]**

**3)** Explain why the angle used in $\frac{1}{2}ab\sin C$ must be the angle between sides $a$ and $b$.

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**[Question 3: 2 marks]**

**4)** Kim says that a triangle with two sides of 5 cm has a bigger area with an included angle of 150° than with 90°. Show that she is wrong.

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**[Question 4: 2 marks]**

**5)** Explain why triangles with sides 4 cm and 7 cm and included angles 30° or 150° have the same area.

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**[Question 5: 2 marks]**

**6)** Show that an equilateral triangle with side $a$ has area $\frac{\sqrt{3}}{4}a^2$

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**[Question 6: 3 marks]**

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**7)** Two sides of a triangle are fixed. Which included angle gives the largest area? Explain.

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**[Question 7: 2 marks]**

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**8)** Show that a parallelogram with sides 6 cm and 10 cm and an angle of 30° has area 30 $cm^2$.

**[Question 8: 2 marks]**

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**9)** A triangle has sides 8 cm and 10 cm and area 20 $cm^2$. Show that the included angle is 30° or 150°.

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**[Question 9: 2 marks]**

**End of paper – 20 marks in total**

# Answers and brief mark scheme

| Q | Answer | Method / notes |
|---|---|---|
| 1 | Shown | Draw the height to side $a$ (M1); $h = b \sin C$ (M1); $\frac{1}{2}ah$ (C1). |
| 2 | Shown | $\frac{1}{2} \times 48 \times 0.5$ (M1); C1. |
| 3 | Explanation | The formula uses the height made by that angle (C2). |
| 4 | Shown | 6.25 and 12.5 (M1); C1. |
| 5 | Explanation | $\sin 150° = \sin 30°$ (C2). |
| 6 | Shown | $\frac{1}{2}a^2 \sin 60°$ (M1); $\sin 60° = \frac{\sqrt{3}}{2}$ (M1); C1. |
| 7 | 90° | $\sin C$ is largest, 1, at 90° (C2). |
| 8 | Shown | $6 \times 10 \times \sin 30°$ (M1); C1. |
| 9 | Shown | $\sin C = 0.5$ (M1); two angles (C1). |