HIGHER ONLY

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Name ..... Class .....

A20: Iteration

Reason with iterative methods and change of sign

20 marks · 25 minutes · Calculator allowed

1) Show that $x^3 + x - 4 = 0$ has a solution between $x = 1$ and $x = 2$

[Question 1: 2 marks]

2) Show that $x^3 + x - 4 = 0$ can be rearranged to give $x = \sqrt[3]{4 - x}$

[Question 2: 2 marks]

3) Explain why a change of sign between two values shows there is a solution between them.

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[Question 3: 1 marks]

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4) Use $x_{n+1} = \sqrt[3]{4 - x_n}$ with $x_0 = 1$ to find x_3 to 4 decimal places. Explain what x_3 represents.

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[Question 4: 3 marks]

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5) Show that the solution of $x^3 + x - 4 = 0$ is 1.38 to 2 decimal places.

[Question 5: 3 marks]

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6) Explain why the iteration $x_{n+1} = 4 - x_n^3$ with $x_0 = 1$ does not help to find the solution.

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[Question 6: 2 marks]

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7) Sam uses $x_{n+1} = \frac{2}{x_n}$ with $x_0 = 1$ to solve $x^2 = 2$. He gets $1, 2, 1, 2, \ldots$ Explain what has happened.

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[Question 7: 2 marks]

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8) Explain why it is sensible to stop iterating when two values agree to the required accuracy.

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[Question 8: 2 marks]

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9) Show that $x = \frac{1}{2}\left(x + \frac{2}{x}\right)$ is a rearrangement of $x^2 = 2$

[Question 9: 3 marks]

End of paper – 20 marks in total

Answers and brief mark scheme

Q	Answer	Method / notes
1	Shown	$f(1) = -2$, $f(2) = 6$ (M1); change of sign (C1).
2	Shown	$x^3 = 4 - x$ (M1); cube root (C1).
3	Explanation	The graph is continuous so it must cross zero (C1).
4	1.3808	1.4422, 1.3676 (M1); 1.3808 (A1); an estimate of the solution (C1).
5	Shown	$f(1.375) = -0.025$ (M1); $f(1.385) = 0.042$ (M1); change of sign (C1).
6	Explanation	Values 3, -23, ... move away (diverge) (C2).
7	Explanation	It oscillates (C1); does not converge to $\sqrt{2}$ (C1).
8	Explanation	Further values will not change the answer at that accuracy (C2).
9	Shown	$2x = x + \frac{2}{x}$ (M1); $x = \frac{2}{x}$ (M1); $x^2 = 2$ (C1).