FOUNDATION & HIGHER

HIGHER EXTENDS

Name ..... Class .....

A6: Algebraic proof

Construct algebraic arguments and proofs

20 marks · 25 minutes · Non-calculator

1) Prove that $(n + 1)^2 - (n - 1)^2 \equiv 4n$

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[Question 1: 2 marks]

2) Prove that the sum of three consecutive integers is always a multiple of 3

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[Question 2: 2 marks]

3) Prove that $(2n + 1)^2 - 1$ is always a multiple of 8

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[Question 3: 3 marks]

4) Explain the difference between $3x + 1 = 7$ and $3(x + 1) \equiv 3x + 3$.

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[Question 4: 2 marks]

5) Prove that the product of two odd numbers is always odd.

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[Question 5: 2 marks]

6) $(x + a)^2 + b \equiv x^2 + 6x + 13$. Show that $a = 3$ and $b = 4$

[Question 6: 2 marks]

7) Prove that the square of any even number is a multiple of 4

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[Question 7: 2 marks]

8) Sam says $n^2 + n + 41$ is always prime. Use $n = 40$ to show he is wrong.

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[Question 8: 2 marks]

9) Prove that the difference between the squares of two consecutive integers is always odd.

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[Question 9: 3 marks]

End of paper – 20 marks in total

Answers and brief mark scheme

Q	Answer	Method / notes
1	Proof	$n^2 + 2n + 1 - (n^2 - 2n + 1)$ (M1); $4n$ (C1).
2	Proof	$n + (n + 1) + (n + 2) = 3n + 3$ (M1); $= 3(n + 1)$ (C1).
3	Proof	$4n^2 + 4n$ (M1); $4n(n + 1)$ (M1); $n(n + 1)$ is even so multiple of 8 (C1).
4	Explanation	Equation is true for one value; identity is true for all values (C2).
5	Proof	$(2a + 1)(2b + 1) = 4ab + 2a + 2b + 1$ (M1); $2(...) + 1$ is odd (C1).
6	Shown	$x^2 + 2ax + a^2 + b$ (M1); $a = 3$, $b = 13 - 9 = 4$ (C1).
7	Proof	$(2n)^2 = 4n^2$ (M1); C1.
8	Counter-example	$1600 + 40 + 41 = 1681$ (M1); $= 41^2$, not prime (C1).
9	Proof	$(n + 1)^2 - n^2$ (M1); $= 2n + 1$ (M1); odd (C1).