HIGHER ONLY

HIGHER TIER

Name ..... Class .....

A20: Iteration

Find approximate solutions to equations using iteration

20 marks · 25 minutes · Calculator allowed

1) Show that the equation $x^3 - 3x - 5 = 0$ has a solution between $x = 2$ and $x = 3$

[Question 1: 2 marks]

2) Show that $x^3 - 3x - 5 = 0$ can be rearranged to give $x = \sqrt[3]{3x + 5}$

[Question 2: 1 marks]

3) Use the iteration formula $x_{n+1} = \sqrt[3]{3x_n + 5}$ with $x_0 = 2$

(a) Find the values of x_1, x_2 and x_3. Give each to 4 decimal places.

x_1 = ...

x_2 = ...

x_3 = ...

(3)

(b) Keep going to find the solution correct to 3 decimal places.

.....

(2)

[Question 3: 5 marks]

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4) $u_{n+1} = 3 - \dfrac{1}{u_n}$ and $u_1 = 2$. Find u_2 and u_3.

u_2 = ... u_3 = ...

[Question 4: 2 marks]

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5) Show that $x^3 + 2x - 5 = 0$ has a solution between $x = 1$ and $x = 2$

[Question 5: 2 marks]

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6) Show that $x^3 + 2x - 5 = 0$ can be rearranged to give $x = \sqrt[3]{5 - 2x}$

[Question 6: 1 marks]

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7) Use $x_{n+1} = \sqrt[3]{5 - 2x_n}$ with $x_0 = 1.5$

(a) Find x_1, x_2 and x_3 to 4 decimal places.

x_1 = ...

x_2 = ...

x_3 = ...

(3)

(b) Keep going to find the solution correct to 2 decimal places.

.....

(2)

[Question 7: 5 marks]

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8) $u_{n+1} = \frac{1}{2}\left(u_n + \frac{6}{u_n}\right)$ and $u_1 = 2$. Find u_2 and u_3.

u_2 = ... u_3 = ...

[Question 8: 2 marks]

End of paper – 20 marks in total

Answers and brief mark scheme

Q	Answer	Method / notes
1	Shown	$2^3 - 6 - 5 = -3$ and $3^3 - 9 - 5 = 13$ (M1); change of sign, so a solution lies between 2 and 3 (A1).
2	Shown	$x^3 = 3x + 5$, then take the cube root (B1).
3	(a) 2.2240, 2.2684, 2.2770 (b) 2.279	(a) B1 each. (b) Further iterations, e.g. 2.2786, 2.2789, 2.2790 (M1); 2.279 (A1).
4	$u_2 = 2.5$, $u_3 = 2.6$	B1 each.
5	Shown	$1 + 2 - 5 = -2$ and $8 + 4 - 5 = 7$ (M1); change of sign so a root lies between (A1).
6	Shown	$x^3 = 5 - 2x$, then cube root (B1).
7	(a) 1.2599, 1.3536, 1.3186 (b) 1.33	(a) B1 each. (b) Further values, e.g. 1.3319, 1.3269, 1.3288 (M1); 1.33 (A1).
8	$u_2 = 2.5$, $u_3 = 2.45$	B1 each.