GCSE HIGHER

ITERATION

9 WORKSHEETS · 8 QUESTIONS EACH · ANSWERS INCLUDED

CONTENTS

APPROXIMATE SOLUTIONS TO EQUATIONS USING ITERATION

1. Showing a solution lies between two values — PAGE 2
2. First three iterations — PAGE 3
3. Estimating a solution with an iteration formula — PAGE 4
4. Iterating to a required accuracy — PAGE 5
5. Checking the accuracy of an estimate — PAGE 6
6. Finding the equation an iteration formula solves — PAGE 8
7. Putting it all together — PAGE 9

ALGEBRAIC MANIPULATION

8. Rearranging into a root formula — PAGE 11

REARRANGE FORMULAE TO CHANGE THE SUBJECT

9. Rearranging into a fraction formula — PAGE 12

Answers — PAGE 13

SHOWING A SOLUTION LIES BETWEEN TWO VALUES

Rearrange so one side is 0 if you need to. Substitute both values: a change of sign shows that a solution lies between them. You may use a calculator.

1 Show that a solution of this equation lies between x = 1 and x = 2

$x^3 + 3x - 9 = 0$

2 Show that a solution of this equation lies between x = 2 and x = 3

$x^3 - 4x - 2 = 0$

3 Show that a solution of this equation lies between x = −1 and x = 0

$x^3 + 5x + 3 = 0$

4 Show that a solution of this equation lies between x = 0 and x = 1

$x^3 - 6x + 1 = 0$

5 Show that a solution of this equation lies between x = 2 and x = 3

$x^3 = 4x + 7$

6 Show that a solution of this equation lies between x = 1.6 and x = 2

$x^4 - 3x - 6 = 0$

7 Show that a solution of this equation lies between x = 1.8 and x = 1.9

$x^3 + x^2 = 10$

8 Show that a solution of this equation lies between x = 2.1 and x = 2.2

$x^5 + x = 50$

FIRST THREE ITERATIONS

Start with the value of x_0 given. Put it into the formula to get x_1, then put x_1 in to get x_2, and so on. Give each value to 4 decimal places. You may use a calculator.

1 $x_{n+1} = \sqrt[3]{9 - x_n}$

$x_0 = 2$

Answer

2 $x_{n+1} = \dfrac{5}{x_n^{\,2} + 3}$

$x_0 = 1$

Answer

3 $x_{n+1} = \sqrt[3]{3x_n + 5}$

$x_0 = 2$

Answer

4 $x_{n+1} = 2 + \dfrac{1}{x_n^{\,2}}$

$x_0 = 2$

Answer

5 $x_{n+1} = \sqrt{4 + \dfrac{3}{x_n}}$

$x_0 = 3$

Answer

6 $x_{n+1} = \sqrt[3]{2x_n - 6}$

$x_0 = -2$

Answer

7 $x_{n+1} = \sqrt[4]{3x_n + 7}$

$x_0 = 1.5$

Answer

8 $x_{n+1} = \dfrac{x_n^{\,3} + 4}{10}$

$x_0 = 0.5$

Answer

ESTIMATING A SOLUTION WITH AN ITERATION FORMULA

Use the iteration formula the number of times asked, starting with x_0. Keep full calculator values (use the ANS key), and round only your final answer. You may use a calculator.

1 Use the formula three times. Give your answer to 2 decimal places.

$x_{n+1} = \sqrt[3]{20 - x_n}$

$x_0 = 3$

$x \approx$

2 Use the formula three times. Give your answer to 3 decimal places.

$x_{n+1} = \sqrt{7 - \frac{2}{x_n}}$

$x_0 = 2$

$x \approx$

3 Use the formula three times. Give your answer to 3 decimal places.

$x_{n+1} = 3 + \frac{2}{x_n^{\,2}}$

$x_0 = 3$

$x \approx$

4 Use the formula four times. Give your answer to 2 decimal places.

$x_{n+1} = \sqrt[3]{3x_n - 8}$

$x_0 = -2$

$x \approx$

5 Use the formula three times. Give your answer to 4 decimal places.

$x_{n+1} = \sqrt[3]{\frac{15 - x_n}{3}}$

$x_0 = 1.5$

$x \approx$

6 Use the formula four times. Give your answer to 3 decimal places.

$x_{n+1} = \sqrt{\frac{10}{x_n + 3}}$

$x_0 = 1$

$x \approx$

7 Use the formula twice. Give your answer to 4 decimal places.

$x_{n+1} = 5 - \frac{1}{x_n}$

$x_0 = 4$

$x \approx$

8 Use the formula three times. Give your answer to 3 decimal places.

$x_{n+1} = \sqrt[4]{2x_n^{\,2} + 11}$

$x_0 = 2$

$x \approx$

ITERATING TO A REQUIRED ACCURACY

Keep using the formula until two values in a row round to the same number. That rounded value is the solution to that accuracy. You may use a calculator.

1 Iterate until two values agree to 3 decimal places.

$x_{n+1} = \sqrt[3]{4x_n + 1}$

$x_0 = 2$

$x \approx$ ____

2 Iterate until two values agree to 2 decimal places.

$x_{n+1} = \dfrac{4}{x_n + 2}$

$x_0 = 1$

$x \approx$ ____

3 Iterate until two values agree to 3 decimal places.

$x_{n+1} = \sqrt[3]{35 - x_n}$

$x_0 = 3$

$x \approx$ ____

4 Iterate until two values agree to 2 decimal places.

$x_{n+1} = \dfrac{10}{\sqrt{x_n + 4}}$

$x_0 = 3$

$x \approx$ ____

5 Iterate until two values agree to 2 decimal places.

$x_{n+1} = \sqrt[3]{2x_n - 9}$

$x_0 = -2$

$x \approx$ ____

6 Iterate until two values agree to 2 decimal places.

$x_{n+1} = \sqrt[3]{7x_n + 3}$

$x_0 = 3$

$x \approx$ ____

7 Iterate until two values agree to 3 decimal places.

$x_{n+1} = 3 - \dfrac{1}{x_n^2}$

$x_0 = 2.5$

$x \approx$ ____

8 Iterate until two values agree to 2 decimal places.

$x_{n+1} = \sqrt[3]{x_n^2 + 9}$

$x_0 = 2.5$

$x \approx$ ____

CHECKING THE ACCURACY OF AN ESTIMATE

To show that a value is correct to some accuracy, substitute the lower and upper bounds of the value: a change of sign shows that a solution lies between them. You may use a calculator.

1 Nia uses iteration and gets $x = 2.2$ as an estimate for a solution of $x^3 - 3x - 4 = 0$.
Substitute 2.2 into $x^3 - 3x - 4$ and comment on the accuracy of her estimate.

Answer

2 A solution of $x^3 + x^2 - 7 = 0$ is $x = 1.63$ correct to 2 decimal places.
By substituting the values 1.625 and 1.635, show that this is true.

3 Show that $x = -0.56$ is a solution of $x^3 + 5x + 3 = 0$ correct to 2 decimal places.

4 Ravi says that $x^3 - 7x + 2 = 0$ has a solution $x = 0.3$, correct to 1 decimal place.
Use the values 0.25 and 0.35 to decide whether Ravi is right.

Answer

CHECKING THE ACCURACY OF AN ESTIMATE (CONTINUED)



5 Lauren says that $x^3 - 5x - 1 = 0$ has a solution between 2 and 3 which is 2.4 correct to 1 decimal place.
Is Lauren right? Substitute 2.35 and 2.45 to decide.

Answer

6 Show that $x^3 + 4x - 15 = 0$ has a solution $x = 1.936$ correct to 3 decimal places.

7 Ben iterates and finds that a solution of $x^3 - 2x^2 - 3 = 0$ is 2.486 correct to 3 decimal places.
Show that Ben's value is correct.

8 Mia and Tom each estimate a solution of $x^3 + x - 20 = 0$. Mia gets 2.59 and Tom gets 2.6.
By substituting both values into $x^3 + x - 20$, decide whose estimate is more accurate.

Answer

FINDING THE EQUATION AN ITERATION FORMULA SOLVES

Each rearranged formula comes from an equation $x^3 + ax + b = 0$, where a and b are integers. Get rid of the root or fraction, then collect everything on one side to find a and b. You may use a calculator.

1 $x = \sqrt[3]{6 - 4x}$

Answer

2 $x = \sqrt[3]{9 + 2x}$

Answer

3 $x = \sqrt[3]{5x - 1}$

Answer

4 $x = \dfrac{7}{x^2 + 3}$

Answer

5 $x = \dfrac{x^3 + 8}{6}$

Answer

6 $x = \sqrt{8 - \dfrac{3}{x}}$

Answer

7 $x = -\sqrt[3]{7x + 2}$

Answer

8 This one has the form $x^3 + ax^2 + b = 0$

$x = 3 - \dfrac{2}{x^2}$

Answer

PUTTING IT ALL TOGETHER



Show the change of sign, then the rearrangement, then iterate. Write down each value of x_n that you find. You may use a calculator.

1 (a) Show that a solution of $x^3 - 5x - 3 = 0$ lies between $x = 2$ and $x = 3$.
(b) Show that the equation can be rearranged to give the iteration formula below.
(c) Use the formula three times, starting with $x_0 = 2.5$, to find an estimate for the solution correct to 3 decimal places.

$$x_{n+1} = \sqrt[3]{5x_n + 3}$$

(c) $x \approx$

2 (a) Show that a solution of $x^3 + 3x^2 - 60 = 0$ lies between $x = 3$ and $x = 4$.
(b) Show that the equation can be rearranged to give the iteration formula below.
(c) Use the formula three times, starting with $x_0 = 3$, to find an estimate for the solution correct to 3 decimal places.

$$x_{n+1} = \sqrt{\frac{60}{x_n + 3}}$$

(c) $x \approx$

3 A cuboid has a square base with sides of length x cm. Its height is 2 cm more than the length of a side of the base. Its volume is 50 cm^3.
(a) Write an equation for x and show that it has a solution between 3 and 4.
(b) Show that your equation can be rearranged to give the formula below.
(c) Starting with $x_0 = 3$, use the formula to find the solution correct to 2 decimal places.

$$x_{n+1} = \sqrt{\frac{50}{x_n + 2}}$$

(c) $x \approx$

4 Kemi thinks of a number, x. The cube of her number is 9 more than 3 times her number.
(a) Write an equation for x and show that it has a solution between 2 and 3.
(b) Show that the equation can be rearranged to give the formula below.
(c) Use the formula three times, starting with $x_0 = 2$, to find an estimate for the solution correct to 3 decimal places.

$$x_{n+1} = \sqrt[3]{3x_n + 9}$$

(c) $x \approx$

PUTTING IT ALL TOGETHER (CONTINUED)



5 (a) Show that a solution of $x^3 - 3x^2 - 2 = 0$ lies between $x = 3$ and $x = 4$.
(b) Show that the equation can be rearranged to give the iteration formula below.
(c) Use the formula three times, starting with $x_0 = 3$, to find an estimate for the solution correct to 3 decimal places.

$$x_{n+1} = 3 + \frac{2}{x_n^{\,2}}$$

(c) $x \approx$

6 (a) Show that a solution of $2x^3 - 11x + 4 = 0$ lies between $x = 0$ and $x = 1$.
(b) Show that the equation can be rearranged to give the iteration formula below.
(c) Use the formula three times, starting with $x_0 = 0.4$, to find an estimate for the solution correct to 4 decimal places.

$$x_{n+1} = \frac{2x_n^{\,3} + 4}{11}$$

(c) $x \approx$

7 A rectangle has sides of length x cm and $(x^2 - 4)$ cm. Its area is 20 cm^2.
(a) Write an equation for x and show that it has a solution between 3 and 4.
(b) Show that your equation can be rearranged to give the formula below.
(c) Starting with $x_0 = 3$, use the formula to find the solution correct to 2 decimal places.

$$x_{n+1} = \sqrt[3]{4x_n + 20}$$

(c) $x \approx$

8 (a) Show that a solution of $x^3 + 6x = 15$ lies between $x = 1$ and $x = 2$.
(b) Show that the equation can be rearranged to give the iteration formula below.
(c) Use the formula four times, starting with $x_0 = 1.5$, to find an estimate for the solution correct to 2 decimal places.

$$x_{n+1} = \frac{15}{x_n^{\,2} + 6}$$

(c) $x \approx$

REARRANGING INTO A ROOT FORMULA



Show that each equation can be rearranged to give the formula under it. Get the power of x on its own, then take the root of both sides. You may use a calculator.

1 $x^3 + 4x - 9 = 0$

$x = \sqrt[3]{9 - 4x}$

2 $x^3 - 6x = 2$

$x = \sqrt[3]{6x + 2}$

3 $2x^3 + 3x - 10 = 0$

$x = \sqrt[3]{\frac{10 - 3x}{2}}$

4 $x^4 - 3x^2 - 8 = 0$

$x = \sqrt[4]{3x^2 + 8}$

5 $x^3 = 5(x + 2)$

$x = \sqrt[3]{5x + 10}$

6 $x(x^2 + 3) = 11$

$x = \sqrt[3]{11 - 3x}$

7 $x^5 + 2x - 30 = 0$

$x = \sqrt[5]{30 - 2x}$

8 $3x^3 - 4x^2 + 5 = 0$

$x = \sqrt[3]{\frac{4x^2 - 5}{3}}$

REARRANGING INTO A FRACTION FORMULA



Show that each equation can be rearranged to give the formula under it. Factorise out x (or x^2), or divide both sides by a power of x. You may use a calculator.

1 $x^3 + 5x - 8 = 0$

$$x = \frac{8}{x^2 + 5}$$

2 $x^3 - 2x - 7 = 0$

$$x = \frac{7}{x^2 - 2}$$

3 $x^3 + 2x^2 - 15 = 0$

$$x = \sqrt{\frac{15}{x + 2}}$$

4 $x^3 - 4x^2 - 3 = 0$

$$x = 4 + \frac{3}{x^2}$$

5 $2x^3 + x - 12 = 0$

$$x = \frac{12}{2x^2 + 1}$$

6 $x^3 - 9x + 4 = 0$

$$x = \sqrt{9 - \frac{4}{x}}$$

7 $x^4 + 2x - 11 = 0$

$$x = \frac{11}{x^3 + 2}$$

8 $x^3 + 6x^2 - 5 = 0$

$$x = -6 + \frac{5}{x^2}$$

ANSWERS

1. Showing a solution lies between two values

1. –5 and 5: sign change
2. –2 and 13: sign change
3. –3 and 3: sign change
4. 1 and –4: sign change
5. –7 and 8: sign change
6. –4.2464 and 4: sign change
7. –0.928 and 0.469: sign change
8. –7.059 and 3.736: sign change

2. First three iterations

1. 1.9129, 1.9208, 1.9201
2. 1.2500, 1.0959, 1.1902
3. 2.2240, 2.2684, 2.2770
4. 2.2500, 2.1975, 2.2071
5. 2.2361, 2.3112, 2.3017
6. –2.1544, –2.1764, –2.1795
7. 1.8415, 1.8812, 1.8857
8. 0.4125, 0.4070, 0.4067

3. Estimating a solution with an iteration formula

1. $x \approx 2.59$
2. $x \approx 2.489$
3. $x \approx 3.196$
4. $x \approx -2.49$
5. $x \approx 1.6450$
6. $x \approx 1.492$
7. $x \approx 4.7895$
8. $x \approx 2.112$

4. Iterating to a required accuracy

1. $x \approx 2.115$
2. $x \approx 1.24$
3. $x \approx 3.169$
4. $x \approx 3.62$
5. $x \approx -2.40$
6. $x \approx 2.84$
7. $x \approx 2.879$
8. $x \approx 2.47$

5. Checking the accuracy of an estimate

1. 0.048, close to 0, so accurate
2. –0.0684 and 0.0439: sign change
3. –0.0054 and 0.0540: sign change
4. Yes: 0.266 and –0.407, sign change
5. No: 0.228 and 1.456 are both positive
6. –0.0073 and 0.0079: sign change
7. –0.0007 and 0.0079: sign change
8. Mia: –0.036 is closer to 0 than 0.176

ANSWERS (CONTINUED)

6. Finding the equation an iteration formula solves

1. $a = 4, b = -6$
2. $a = -2, b = -9$
3. $a = -5, b = 1$
4. $a = 3, b = -7$
5. $a = -6, b = 8$
6. $a = -8, b = 3$
7. $a = 7, b = 2$
8. $a = -3, b = 2$

7. Putting it all together

1. -5 and 9; $x \approx 2.491$
2. -6 and 52; $x \approx 3.131$
3. -5 and 46; $x \approx 3.12$
4. -7 and 9; $x \approx 2.552$
5. -2 and 14; $x \approx 3.196$
6. 4 and -5; $x \approx 0.3731$
7. -5 and 28; $x \approx 3.20$
8. -8 and 5; $x \approx 1.66$

8. Rearranging into a root formula

1. $x^3 = 9 - 4x$, so $x = \sqrt[3]{9 - 4x}$
2. $x^3 = 6x + 2$, so $x = \sqrt[3]{6x + 2}$
3. $2x^3 = 10 - 3x$, so $x = \sqrt[3]{\frac{10 - 3x}{2}}$
4. $x^4 = 3x^2 + 8$, so $x = \sqrt[4]{3x^2 + 8}$
5. $x^3 = 5x + 10$, so $x = \sqrt[3]{5x + 10}$
6. $x^3 + 3x = 11$, so $x = \sqrt[3]{11 - 3x}$
7. $x^5 = 30 - 2x$, so $x = \sqrt[5]{30 - 2x}$
8. $3x^3 = 4x^2 - 5$, so $x = \sqrt[3]{\frac{4x^2 - 5}{3}}$

9. Rearranging into a fraction formula

1. $x(x^2 + 5) = 8$
2. $x(x^2 - 2) = 7$
3. $x^2(x + 2) = 15$
4. $x^3 = 4x^2 + 3$, divide by x^2
5. $x(2x^2 + 1) = 12$
6. $x^2 = 9 - \frac{4}{x}$
7. $x(x^3 + 2) = 11$
8. $x^3 = 5 - 6x^2$, divide by x^2